Midterm: Eng. 0138 Advanced Control II

February 26, 2003.
8:30 AM to 9:30 AM
Closed Book exam.
Programmable calculators are not allowed.

Q-1 (8 marks)
Consider the following system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = [1 \quad 0]$$

a) Write the system in controllable canonical form, observable canonical form and diagonal canonical form or Jordan canonical form.

b) Compute e^{At}.

Q-2 (17 marks)
Consider the system defined by:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where,

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = [1 \quad 0]$$

a) Check the state controllability and observability of the system.
b) Using the state feedback control $u = -Kx + lr$, determine K and l such that the closed loop system will have:
   i) Zero-steady-state error for a unit-step input
   ii) 10% overshoot and 10 seconds settling time.

c) Give the block diagram of the observer-based state feedback controller.

Percent overshoot: $\% OS = 100 \exp(\frac{-\xi\pi}{\sqrt{1-\xi^2}})$

Settling time at 2%: $T_s \approx \frac{4}{\xi\omega_n}$

$y = Cx$ $\quad \begin{bmatrix} \\ -3 & -4 \end{bmatrix}, \; \begin{bmatrix} \\ 1 \end{bmatrix}$

$C = [1, 0]$

Step 1
find transfer function.

$\dot{X} = Ax + Bu \Rightarrow sX(s) = Ax + Bu$
$X(s)(sI - A) = Bu$
$X(s) = (sI-A)^{-1} Bu$

$Y = Cx$
$Y(s) = C(sI-A)^{-1} B \cdot U(s)$
$\frac{Y(t)}{U(t)} = C(SI-A)^{-1} B$

$(s+3)(s+$
$Det = s^2+4s+3$

Sub $\begin{pmatrix} s & -1 \\ 3 & s+4 \end{pmatrix}^{-1} \Rightarrow \frac{1}{(s+3)(s+1)} \begin{pmatrix} s+4 & 1 \\ -3 & s \end{pmatrix}$

$T.F. = [1 \; 0]\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix}\right]^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= (1 \; 0)\begin{pmatrix} s+4 & 1 \\ -3 & s \end{pmatrix} = [(s+4) \; 1]$

$1\times2 \quad 2\times2$

$= [(s+4) \; 1]\begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \frac{1}{(s+3)(s+1)} = \frac{Y(s)}{U(s)}$

$Y'' + 4Y' + 3Y = U(s)$

$X_1(t) = Y(t)$ $\qquad \dot{X}_2 = -4X_2 - 3X_1 + U(s)$
$\dot{X}_1 = \dot{Y}_1 = X_2$ $\qquad \dot{X}_1 = X_2$
$\dot{X}_2 = \ddot{Y}$

C_c $\quad A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [1, 0] \quad D = 0$

Observ...
$A_{Oc} = A^T_{Cc}$ $\qquad B_{Oc} = C_c^T$ $\qquad C_{Oc} = B_c^T = [0 \; 1] \quad D = 0$
$A_{Oc} = \begin{bmatrix} 0 & -3 \\ 1 & -4 \end{bmatrix}$ $\qquad B_{Oc} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$e^{a21} = \frac{-3}{(s+3)(s+1)}$

$Y(s)\ (s+3) \Big|_{s\to -3} \quad \frac{-3}{-2} = \frac{3}{2} = A$

$\frac{3}{2} \cdot \frac{1}{(s+3)} - \frac{3}{2}\frac{1}{(s+1)}$

$Y(s)\ (s+1) \Big|_{s\to -1} \quad \frac{-3}{2} \Rightarrow \frac{-3}{2}$

$e^{a22} = \frac{s}{(s+3)(s+1)}$

$Y(s)\ (s+3) \Big|_{s\to -3} = \frac{+3}{+2} = A \quad \frac{A}{(s+3)} + \frac{B}{(s+1)} = s$

$\frac{3}{2} \cdot \frac{1}{(s+3)} - \frac{1}{2} \cdot \frac{1}{(s+1)}$

$Y(s)\ (s+1) \Big|_{s\to -1} \quad \frac{-1}{2} = B$

$$e^{at} = \begin{bmatrix} \left(-\frac{1}{2}\cdot\frac{1}{(s+3)} + \frac{3}{2}\frac{1}{(s+1)}\right) & \left(\frac{1}{2(s+3)} - \frac{1}{2(s+1)}\right) \\ \left(\frac{3}{2(s+3)} - \frac{3}{2(s+1)}\right) & \left(\frac{3}{2(s+3)} - \frac{1}{2(s+1)}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2}e^{-3t} + \frac{3}{2}e^{-t} \\ \frac{3}{2}e^{-3t} + \frac{3}{2}e^{-t} \end{bmatrix}$$

controllers

a) $\mathcal{C} = [B \quad AB \quad A^{n-1}B]$
$= [B \quad AB]$

$AB = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Det $\Rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$

$0 \neq -1$ ✓

Obser

$O = \begin{bmatrix} C \\ CA \\ CA^{n-1} \end{bmatrix} \Rightarrow$

2x2

$CA = [1\ 0]\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = [0\ 1]$

Det $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \neq 0$ ✓

b) $\dot{x} = Ax + Bu \qquad u = -Kx + lr$
$y = Cx$

$\dot{x} = Ax - BKx + Blr$
$\dot{x} = x(A - BK) + Blr$
$x(sI - (A-BK)) = Blr$
$x = (sI - (A-BK))^{-1} Blr$
$y(t) = C[sI - (A-BK)]^{-1} Blr$?

$\lim_{s\to 0} y(t) = \frac{1}{s} C[-(A-BK)^{-1}] Bls$

$1 = C[-(A-BK)^{-1}]Bl$

$l = [C(-A+BK)^{-1}B]^{-1}$

find K?

$TF = \frac{1}{(s+3)(s+1)}$ $\frac{u\,C_1}{(s+3)} + \frac{u\,C_2}{(s+1)} = 1$

$A = \begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix}$ $(s+1)\Big|_{s\to -1} = \frac{1}{2} = C_2$ $C_1 \Rightarrow s+3\Big|_{s\to -3} \frac{1}{-2}$

$C_1 = \frac{1}{2}$ $C_2 = \frac{-1}{2}$ $C = [1/2 \ , -1/2]$

$B \Rightarrow$ $X_1 = \frac{1}{s+3} u_{(t)} \Rightarrow X(s) + 3X = 1\, u(t)$

$B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $D = 0$

(7) Jordan $\frac{1}{(s+3)(s+1)}$

$X(t) = e^{At} X(0) + \int_0^t e^{A(t-\tau)} B u \, d\tau$

$e^{At} = \mathcal{L}^{-1}\left((sI - A)^{-1} \right)$

$e^{At} = \mathcal{L}^{-1}\left(\left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \right]^{-1} \right)$

Det $= s^2 + 4s + 3$

$\Rightarrow \begin{bmatrix} s & -1 \\ +3 & s+4 \end{bmatrix}^{-1} = \begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix} \frac{1}{(s+3)(s+1)}$

$e^{At} \Rightarrow \mathcal{L}^{-1} \begin{bmatrix} \frac{s+4}{(s+3)(s+1)} & \frac{1}{(s+3)(s+1)} \\ \frac{-3}{(s+3)(s+1)} & \frac{s}{(s+3)(s+1)} \end{bmatrix}$

$e^{At} \Rightarrow s+4 = \frac{A}{(s+3)} + \frac{B}{s+1}$ $(s+3)\, Y(s)\Big|_{s\to -3} \frac{1}{-2} = A$

$\frac{-1}{2}\left(\frac{1}{s+3}\right) + \frac{3}{2}\frac{1}{s+1}$ $(s+1)\, Y(s)\Big|_{s\to -1} \frac{3}{2} = B$

$e^{a_{11}} = \frac{-1}{2}\left(\frac{1}{s+3}\right) + \frac{3}{2}\left(\frac{1}{s+1}\right)$

$e^{a_{12}} = \frac{A}{s+3} + \frac{B}{s+1} = 1$ $(s+3)\, Y(s)\Big|_{s\to -3} \frac{1}{-2} = B$

$e^{a_{12}} = \frac{1}{2}\left(\frac{1}{s+3}\right) + \frac{1}{2}\left(\frac{1}{(s+1)}\right)$ $(s+1)\, Y(s)\Big|_{s\to -1} \frac{1}{2} = A$

$$\mathrm{Det}\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \left[\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} k_1 & k_2 \end{pmatrix}\right]\right]$$

$$\Rightarrow \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \left[\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ k_1 & k_2 \end{pmatrix}\right]$$

$$\Rightarrow \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1-k_1 & 2-k_2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} s & -1 \\ -1+k_1 & s-2+k_2 \end{pmatrix} \Rightarrow$$

$$= (s^2 - 2s + sk_2) + (-1+k_1)$$
$$= s^2 + s(-2+k_2) + (-1+k_1)$$

$$\%OS = 100\, e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \Rightarrow \left(-\ln\frac{10}{100}\right)^2 = \frac{\pi^2\zeta^2}{1-\zeta^2}$$

$$\Rightarrow \quad x^2 - \zeta^2 x^2 = \pi^2\zeta^2$$
$$x^2 = \zeta^2(\pi^2 + x^2)$$
$$\sqrt{\frac{x^2}{\pi^2+x^2}} = \zeta$$
$$\zeta = 0.591155033$$

$$10 = \frac{4}{0.59 \cdot \omega_n}$$
$$\omega_n = 0.676641452$$
$$\omega_n^2 = 0.457843654$$

$$s^2 + 0.8s + 0.457843654 = s^2 + s(-2+k_2) + (-1+k_1)$$

$0.8 = -2 + k_2$ $\qquad$ $-1 + k_1 = 0.4578$

$k_2 = 2.8$ $\qquad$ $k_1 = 1.458$

$$= [1\ 0]\left[\left[\begin{pmatrix} 0 & -1 \\ -1 & -2 \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}[k_1\ k_2]\right]^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right]^{-1}$$

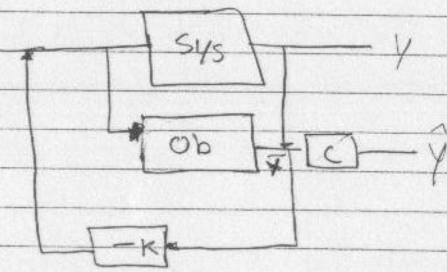
$$\Rightarrow \left[\begin{bmatrix} 0 & -1 \\ -1 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}\right]$$

$$\Rightarrow \begin{pmatrix} 0 & -1 \\ -1+k_1 & -2+k_2 \end{pmatrix}^{-1} \Rightarrow \underset{2\times 2}{\begin{bmatrix} -2+k_2 & 1 \\ -(1+k_1) & 0 \end{bmatrix}}\underset{2\times 1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

$$\Rightarrow \left[\underset{1\times 2}{[1\ 0]}\underset{2\times 1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}\right]^{-1} \Rightarrow 1$$

$L = 1$

c)



Midterm 2003

(1)

(Q1) Given: $\dot{x} = Ax + BU$ $\quad A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [1 \quad 0]$

a) $y = CX$

$y'' + 4y' + 3y = U(s)$

$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$\dot{X}_1 = \dot{Y}_1 = X_2 \quad (Y+3)(Y+1)$

$\dot{X}_2 = \ddot{Y}_1$

$\dot{x}_1 = x_2$

$\dot{x}_2 = -3x_1 - 4x_2 + u$

$\dot{X} = AX + BU \Rightarrow SX(s) = AX(s) + BU \longrightarrow SX(s) - AX(s) = BU$

$X(s)[S - A] = BU$

$X(s)(SI - A) = BU$

$X(s) = (SI - A)^{-1} BU$

Diagonal $\quad T.F. = \dfrac{1}{(S+3)(S+1)} = \dfrac{uC_1}{S+3} + \dfrac{uC_2}{S+1} = 1$

$A = \begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix} \quad C_1 = S+1 \Big|_{S=-1} \dfrac{1}{(-1+3)} = \dfrac{1}{2} \quad C_2 = S+3 \Big|_{S=-3} \dfrac{1}{-3+1} = \dfrac{1}{2}$

$C_1 = 1/2$, $C_2 = -1/2$

$C = [1/2 \quad -1/2]$

$B \Rightarrow X_1 = \dfrac{1}{S+3} u(t)$

$X(s) + 3X =$

$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad D = 0$

$Y(t) = CX$

$Y(t) = C(SI - A)^{-1} BU(t)$

$\boxed{\dfrac{Y(t)}{U(t)} = C(SI - A)^{-1} B}$

$= [1 \quad 0] \left[\begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix} \right]^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= (1 \quad 0) \begin{pmatrix} S & -1 \\ 3 & S+4 \end{pmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underbrace{\underset{1\times 2}{[1 \quad 0]} \underset{2\times 2}{\begin{bmatrix} S+4 & -1 \\ -3 & S \end{bmatrix}}}_{} \underset{2\times 1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} = [1 \quad 0] \begin{bmatrix} 1 \\ S \end{bmatrix} = 1$

Controllable

$C_c = \quad A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [1 \quad 0] \quad D = 0$

$\underset{1\times 2}{\begin{bmatrix} (S+4) & 1 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} = 1 \Rightarrow \dfrac{Y(s)}{U(s)} = \dfrac{1}{(S+3)(S+1)}$

$= \dfrac{1}{S^2 + 4S + 3}$

Observ.

$AO_c = A^T C_c = \begin{bmatrix} 0 & -3 \\ 1 & -4 \end{bmatrix} \quad BO_c = C_c^T \quad CO_c = BC^T = [0 \quad 1]$

$BO_c = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad D = 0$

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(2)

$\det = s(s+4)+3$
$= s^2+4s+3 = (s+3)(s+1)$

Q1 b) $\boxed{e^{At} = \mathcal{L}^{-1}\left[(sI-A)^{-1}\right]}$

$$e^{At} = \mathcal{L}^{-1}\left[\begin{bmatrix} s & 0 \\ 0 & s\end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -4\end{bmatrix}\right]^{-1} = \mathcal{L}^{-1}\begin{bmatrix} s & -1 \\ 3 & s+4\end{bmatrix}^{-1} = \mathcal{L}^{-1}\begin{bmatrix} \frac{s+4}{(s+3)(s+1)} & \frac{1}{(s+3)(s+1)} \\ \frac{3}{(s+3)(s+1)} & \frac{s}{(s+3)(s+1)}\end{bmatrix}$$

(11)

$$\frac{s+4}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1}$$

$$(s+3)Y(s)\Big|_{s=-3} \quad \frac{s+4}{s+1} = \frac{-3+4}{-3+1} = \frac{1}{-2} = A$$

$$= -\frac{1}{2}\left(\frac{1}{s+3}\right) + \frac{3}{2}\left(\frac{1}{s+1}\right)$$

$$(s+1)Y(s)\Big|_{s=-1} \quad \frac{s+4}{s+3} = \frac{-1+4}{-1+3} = \frac{3}{2} = B$$

$$\mathcal{L}^{-1} = -\frac{1}{2}e^{-3t} + \frac{3}{2}e^{-t}$$

(12)

$$\frac{1}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1}$$

$$s+3\Big|_{s=-3} \quad \frac{1}{(s+1)} = \frac{1}{-2} = A$$

$$\mathcal{L}^{-1} = -\frac{1}{2}e^{-3t} + \frac{1}{2}e^{-t}$$

$$s+1\Big|_{s=-1} \quad \frac{1}{(s+3)} = \frac{1}{2} = B$$

(21)

$$\frac{3}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1}$$

$$s+3\Big|_{s=-3} \quad \frac{3}{(s+1)} = \frac{3}{-2} = A$$

$$\mathcal{L}^{-1} = -\frac{3}{2}e^{-3t} + \frac{3}{2}e^{-t}$$

$$s+1\Big|_{s=-1} \quad \frac{3}{(s+3)} = \frac{3}{2} = B$$

(22)

$$\frac{s}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{(s+1)}$$

$$s+3\Big|_{s=-3} \quad \frac{s}{(s+1)} = \frac{-3}{-2} = \frac{3}{2} = A$$

$$s+1\Big|_{s=-1} \quad \frac{s}{s+3} = \frac{-1}{2} = B$$

$$\mathcal{L}^{-1} = \frac{3}{2}e^{-3t} - \frac{1}{2}e^{-t}$$

$a=3$
$b=1$

#16 rule

$$\frac{1}{1-3}\left(1e^{-t} - 3e^{-3t}\right)$$

$$= -\frac{1}{2}\left(e^{-t} - 3e^{-3t}\right)$$

$$e^{At} = \begin{bmatrix} -\frac{1}{2}e^{-3t} + \frac{3}{2}e^{-t} & -\frac{1}{2}e^{-3t} + \frac{1}{2}e^{-t} \\ -\frac{3}{2}e^{-3t} + \frac{3}{2}e^{-t} & \frac{3}{2}e^{-3t} - \frac{1}{2}e^{-t}\end{bmatrix}$$

Midterm 2003 (3)

Q2 a) check state cont. + obser. of system.

procedure cont. : - find $C = [B \;\; AB]$

- det $C \neq 0 \rightarrow$ then good.

$A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}_{2\times2}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2\times1}$ $AB = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$C = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ $\therefore \det C = 0 - 1 = -1 \neq 0 \;\therefore \checkmark$

procedure obser. : - find $O = \begin{bmatrix} C \\ CA \end{bmatrix}$

- $\det(O) \neq 0 \Rightarrow$ then good.

$C = [1 \; 0]$

$CA = [1 \; 0]_{1\times2} \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}_{2\times2} = [0 \; 1]$

$O = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\det(O) = 1 - 0 = 1 \neq 0 \;\therefore \checkmark$

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b) $u = -Kx + lr$, find K and l @ P.O. = 10%, $T_s = 10$sec.

$K = [K_1 \; K_2]$

$100 e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = 10$

$\frac{(-\xi\pi)^2}{1-\xi^2} = [\ln 0.1]^2$

$\xi^2 \cdot \pi^2 - 5.3 + 5.3\xi^2 = 0$

$\xi^2[\pi^2 + 5.3] = 5.3$

$\xi = \sqrt{\frac{5.3}{\pi^2 + 5.3}}$

$\underline{\xi = 0.591}$

$10 = \frac{4}{0.591 \times \omega_n}$

$5.91 \omega_n = 4$

$\underline{\omega_n = 0.677}$

$\omega_n^2 = 0.445$

$\boxed{s^2 + 2\xi\omega_n s + \omega_n^2 = 0}$

$s^2 + 2(0.591)(0.677)s + 0.445 = 0$

$s^2 + 0.8s + 0.445 = 0$

Feb. 8th. Notes.

$\dot{x} = Ax + Bu$ $\quad$ $(u = -Kx + lr)$

$y = Cx$

$\dot{x} = Ax + BKx + lrB$ sub u

$\dot{x} = x(A - BK) + lrB$ factor x

$x[sI - (A - BK)] = Blr$?

$x = [sI - (A - BK)]^{-1} Bl\underline{r}$ $x =$

$y = C[sI - (A - BK)]^{-1} Bl\underline{r}$ sub x into y

$\lim_{s \to 0} = C[-(A - BK)^{-1}]Bl\cancel{r}$ unit step input.

$1 = [C(-A + BK)^{-1}B]Bl$

$l = [C(-A + BK)^{-1}B]^{-1}$

find K?

$[-(A - BK)^{-1}]$

$\det(sI - [A - BK])$

$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \left[\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}_{2\times2} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2\times1} [K_1 \; K_2]_{1\times2}\right] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \left(\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ K_1 & K_2 \end{bmatrix}\right)$

$= \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1-K_1 & 2-K_2 \end{bmatrix} = \begin{bmatrix} s & -1 \\ K_1 - 1 & s + K_2 - 2 \end{bmatrix}$

$\det = s(s + K_2 - 2) - (1 - K_1)$

$= s^2 + sK_2 - 2s - 1 + K_1$

$= s^2 + s(K_2 - 2) + (K_1 - 1)$

$0.8 = K_2 - 2$

$\underline{K_2 = 2.8}$

$0.445 = K_1 - 1$

$\underline{K_1 = 1.445}$

(4)

find ℓ

$$\ell = \left[C(-A+BK)^{-1}B\right]^{-1}$$

$$\ell = \left[\begin{bmatrix}1 & 0\end{bmatrix}\left[\begin{bmatrix}0 & -1\\ -1 & -2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}\begin{bmatrix}K_1 & K_2\end{bmatrix}\right]^{-1}\begin{bmatrix}0\\ 1\end{bmatrix}\right]^{-1}$$

$$\text{adj}\,A = \begin{bmatrix}K_2-2 & 1\\ -K_1+1 & 0\end{bmatrix}$$

$$\ell = \left[\begin{bmatrix}1 & 0\end{bmatrix}\left[\begin{bmatrix}0 & -1\\ -1 & -2\end{bmatrix} + \begin{bmatrix}0 & 0\\ K_1 & K_2\end{bmatrix}\right]^{-1}\begin{bmatrix}0\\ 1\end{bmatrix}\right]^{-1} =$$

$$A^{-1} = \frac{\text{adj}\,A}{\det A}$$

$$\ell = \left[\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}0 & -1\\ K_1-1 & K_2-2\end{bmatrix}^{-1}\begin{bmatrix}0\\ 1\end{bmatrix}\right]^{-1}$$

$$\det A = 0 - (-K_1+1) = K_1 - 1$$

$$\ell = \left[\frac{\begin{bmatrix}1 & 0\end{bmatrix}}{K_1-1}\underbrace{\begin{bmatrix}-2+K_2 & 1\\ 1-K_1 & 0\end{bmatrix}}_{2\times 2}\begin{bmatrix}0\\ 1\end{bmatrix}\right]^{-1}$$

$$\ell = \left[\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{1\times 2}\underbrace{\begin{bmatrix}\frac{K_2-2}{K_1-1} & \frac{1}{K_1-1}\\ \frac{1-K_1}{K_1-1} & 0\end{bmatrix}}_{2\times 2}\begin{bmatrix}0\\ 1\end{bmatrix}\right]^{-1}$$

$$\ell = \left[\underbrace{\begin{bmatrix}\frac{K_2-2}{K_1-1} & \frac{1}{K_1-1}\end{bmatrix}}_{1\times 2}\underbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}_{2\times 1}\right]^{-1} = \left[\frac{1}{K_1-1}\right]^{-1}$$

Sub K_1 in $= 2.4\ldots$

c)

$$u = -Kx$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$\hat{y} = C\hat{x}$$